Nuclear Instruments and Methods in Physics Research A 844 (2017) 62–71

Contents lists available at ScienceDirect

Nuclear Instruments and Methods in Physics Research A

journal homepage: www.elsevier.com/locate/nima

Active lower order mode damping for the four rod LHC crab cavity

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ARTICLE INFO

Keywords:
Active damping
Crab cavity
HL-LHC
LHC

ABSTRACT

The high luminosity upgrade planned for the LHC requires crab cavities to rotate bunches into alignment at the interaction points. They compensate for a crossing angle near 500 μrad. It is anticipated that four crab cavities in succession will be utilized to achieve this rotation on either side of each IP in a local crossing scheme. A crab cavity operates in a dipole mode but always has an accelerating mode that may be above or below the frequency of the operating mode. Crab cavities are given couplers to ensure that unwanted acceleration modes are strongly damped however employing standard practice these unwanted modes will always have some level of excitation. Where this excitation has a random phase it might promote bunch growth and limit beam lifetime. This paper sets out a method for active control of the phase and amplitude of the unwanted lowest accelerating mode in the crab cavities. The paper investigates the level of suppression that can be achieved as a function cavity quality factor and proximity to resonance.

1. Introduction

This paper demonstrates by analysis and modeling the feasibility of applying active damping to the lowest unwanted acceleration mode in crab cavities as would be appropriate for the LHC luminosity upgrade. This paper sets out the configuration and timing enabling a Low Level RF (LLRF) control system to actively damp the unwanted mode.

A novel aspect of this paper is the implementation of a cyclic or multi-valued set point. An unwanted mode must be controlled by RF near its center frequency by manipulation of the I and Q components. Excitation is at the bunch repetition frequency and a designer aims for this to have no harmonic relationship to the unwanted modes. The paper shows how a cyclic or multi-valued set point minimizes control action.

The planned LHC luminosity upgrade [1] will utilize compact crab cavities [2] to adjust the orientation of the proton bunches at certain interaction points (IP) so as to increase luminosity to a defined level that can be maintained throughout the bunch lifetime [3]. Maximum luminosity is achieved when bunches are in perfect alignment. Depending on the luminosity leveling scheme utilized, perfect alignment might not be utilized until the bunch population has been depleted after many hours of operation. For the proposed optics, luminosity would be reduced by a factor of about four when there is no bunch alignment using a crab cavity. The precise reduction factor depends on the level of focusing achieved. The proposal for the luminosity upgrade is to have control of the crabbing angles at interaction points 1 (ATLAS) and 5 (CMS).

A crab cavity is a deflection cavity operated with a 90° phase shift [4] so that a particle at the front of a bunch gets a transverse momentum kick equal and opposite to a particle at the back of a bunch while a particle at the bunch center receives no transverse momentum kick. The overall effect is the application of an apparent rotation to the bunch. In this paper a transverse change in momentum for a bunch or a particle as it passes through a cavity will be referred to as a kick. A kick is the integral of the force with respect to time per unit charge. As protons at the LHC travel close to the speed of light, the kick divided by the velocity of light is a voltage and henceforth all kicks will be expressed as a voltage.

The simplest scheme for controlling crabbing angles is a global scheme as was applied at KEKB [5]. In such a scheme only one crab cavity is required per ring. Once the bunch has a crabbing angle it rotates one way and then the other way with respect to its nominal path as it passes through successive quadrupoles. For a given transverse voltage in the crab cavity the maximum angle of rotation is limited by the focusing properties of the lattice. The lattice is arranged so that bunches have the ideal crabbing angle at the interaction points. For the LHC luminosity upgrade, studies have indicated that having the bunch oscillating about its axis along the entire circumference is unacceptable; for this reason the current proposal is to use a local crabbing scheme [6].

For a local scheme, crab cavities would be located before and after each of the two IPs so that the crab rotation can be removed. Both sets of crab cavities are positioned in a location of relatively high beta so as to minimize the voltage that must be applied in order to get the

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http://dx.doi.org/10.1016/j.nima.2016.11.021
Received 3 August 2016; Received in revised form 30 September 2016; Accepted 8 November 2016
Available online 12 November 2016

appropriate rotation at the IP and to cancel the rotation after the IP.

The highest bunch repetition rate at the LHC is 40.08 MHz for 25 ns operation and 20.04 MHz for 50 ns operation, the crab cavity needs to operate at a multiple harmonic of these frequencies. Crab cavities are currently being designed to operate at 400.8 MHz which is the same frequency as the accelerating RF and is sufficiently low for non linearities of the crab kick along the length of the 80 mm long bunches to be acceptable with respect to machine performance [6].

A crab cavity invariably uses a dipole mode to provide the transverse momentum kick. All RF cavities which admit dipole modes must also admit longitudinal modes. A designer aims for a high R/Q value of the operating dipole mode and low R/Q values for other modes. The R/Q value for each mode is $1/(2\omega C)$, which is half the capacitive impedance and it determines the level of interaction of that mode with bunches passing through the cavity. Here the shunt impedance is taken as the acceleration voltage squared divided by the dissipated power, V^2/P. Crabbing and deflecting cavities designed to operate in a dipole mode will always have one accelerating mode with an R/Q value comparable with the dipole mode's R/Q. Typically this mode has a frequency which is below that of the dipole mode as would be the case for the compact four rod crab cavity [7]. Design optimization of the four rod cavity reduced the R/Q of the low frequency accelerating mode to less than 1/7 of the R/Q of the operating dipole mode. An innovative design for the LHC crab cavity also exists where the acceleration mode frequency is somewhat higher than the operating mode [8]. For this and similar cavities the R/Q of the accelerating mode is between 1/2 and one 1/3 of the R/Q of the operating mode and hence more damping is required.

Section 2 of this paper looks at the level of bunch by bunch excitation that would exist in the Lowest Order Mode (LOM) of the four rod crab cavity when strongly damped with an external Q-factor, Q_e of 100 and for the anticipated LHC bunch structure. This would often be referred to as the sum wake.

Section 3 proposes active damping with a feed forward controller as a method to further reduce longitudinal dispersion of bunches. Feed forward has been demonstrated experimentally on accelerating cavities as a means of compensating beam loading [9], although this paper outlines how such a scheme could be used for compensating excitation of unwanted longitudinal modes in deflecting cavities. Active damping has been investigated previously for mixed higher order modes in a superconducting cavity [10]. The paper claimed some level of success however the damping was not sufficient over a range of modes to warrant implementation at CEBAF. The expected level of damping achievable for the four rod LHC crab cavity is much higher by virtue of the fact that the active damping control system can be optimized to eliminate excitation in a single mode. Damping the acceleration mode of the crab cavity to a Q_e of 100 without compromising the operating mode is technically challenging. It is hoped that the application of active damping will allow the level of passive damping to be reduced.

Section 4 simulates the effectiveness of active damping at eliminating variations in longitudinal acceleration after gaps in the LHC bunch structure. Results presented in this section are again for the case when the acceleration mode is strongly damped with a Q_e of 100. This is the required level of damping in the absence of active damping.

Section 5 firstly considers active damping with the same control parameters used in Section 4 for the case when Q_e is increased to 300. As the quality factor is increased it becomes increasingly unlikely that the acceleration mode could be driven to become resonant. Covering a worst case scenario, this section shows that satisfactory active damping of the accelerating mode can be achieved even when it has moved to become resonant with the bunch repetition frequency.

Section 6 considers active damping performance with moderate detuning and significant measurement errors. After the consideration of measurement errors it is apparent that even a relatively poor estimate for the feed forward term still gives greatly improved damping performance with respect to the case without control.

Calculations and numerical simulations reported in this paper have been obtained by integration of the envelope equations [11] and the model is described in the appendix. The envelope equations are also used to model the output circuit of the power amplifier. This assumes the amplifier has an output cavity or tank circuit as would be the case for all high power, high efficiency amplifiers. Input parameters for the model include measurement errors, latency in the control system, microphonics and bunch charge fluctuations. The feed forward control scheme that has been proposed eliminates issues with latency (time delays). Solutions of the envelope equations with no measurement delays give the required feed forward drive power.

2. Mode excitation with no damping

A cavity mode voltage $V(t)$ can be referenced to its center angular frequency ω in terms of its in phase and quadrature components as

$$V(t) = \Re[(A_r + jA_i)e^{-j\omega t}]. \tag{1}$$

Increments for the in phase and quadrature parts of the phasor induced by a bunch of charge q passing through the cavity with RF phase α are given by

$$\delta A_r = \frac{q\omega}{2}\left(\frac{R}{Q}\right)\cos\alpha \tag{2}$$

and

$$\delta A_i = \frac{q\omega}{2}\left(\frac{R}{Q}\right)\sin\alpha. \tag{3}$$

When the unwanted accelerating mode frequency of a crab cavity is close to a multiple of the bunch repetition frequency then the phase α varies slowly in time and large voltages accumulate in the cavity.

Excitation within a bandwidth is referred to as resonant and the voltage moves in phase with the excitation. For modes with high loaded Q-factors, Q_L, and when a multiple of the bunch repetition frequency is not within several bandwidths of the cavity's natural frequency then the final voltage settles between quadrature and anti-phase to the kick being provided by the bunches. Fig. 1 shows the cavity voltage phase before and after the passage of a bunch when not excited near to resonance; this is the case of most interest as one designs cavities to

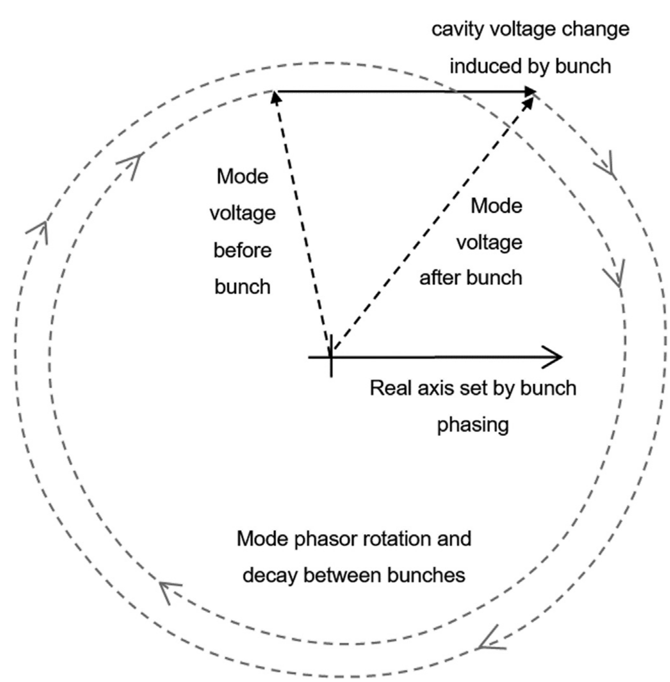

Fig. 1. Off resonant excitation of a mode.

avoid on resonance excitation of unwanted modes. Between bunches the mode phasor rotates and decays to its initial state. Close examination of the phasor diagram reveals the bunch initially sees a small acceleration followed by a stronger deceleration; the voltage has a small decrease followed by a larger increase. This means that the field induced in the mode tends to stretch a bunch; which is undesirable.

In order to limit beam induced accelerating voltages in the crab cavity a coupler is used which extracts power from the unwanted acceleration mode but rejects power from the operating dipole mode. This coupler requires a notch filter if the acceleration mode's frequency is below the dipole mode and a simpler high pass filter if the acceleration mode's frequency is well above the dipole mode frequency.

If conditions allow large voltages to develop in an accelerating mode then depending on the loaded Q factor of the mode and the frequency offset from the operating dipole mode then significant power can be extracted from the beam and this power must exit the cavity through the coupler.

The voltage kick that acts on a bunch is the energy change ΔU of the cavity associated with the voltage increment divided by the bunch charge. From Eqs. (2) and (3) one can show

$$\frac{\Delta U}{q} = A_r \cos\alpha + A_i \sin\alpha + \frac{q\omega}{4}\left(\frac{R}{Q}\right) \tag{4}$$

where $A_r \cos\alpha + A_i \sin\alpha$ is the field in the cavity at the instant that the bunch arrives. From Eq. (4) one sees that it is possible to design an LLRF system that puts a small field in the cavity that accelerates the bunch as it approaches. The field then changes direction as the bunch deposits its image charge. The field then retards the bunch as it leaves. In this way an LLRF system can be designed so that bunches never receive a net voltage kick. With respect to Fig. 1 this would be the case where the mode vectors before and after are symmetrical about the imaginary axis. It should be noted that if the unwanted mode frequency is exactly halfway between resonant frequencies then acceleration is equal to deceleration without a LLRF correction. A phasing which accelerates and then decelerates can stretch the bunch hence optimizing for zero kick is not necessarily the best control strategy for beam lifetime. Whilst this option will be investigated, the paper also investigates strategies where one only aims to give every bunch the same kick; for example, acceleration cavities are usually phased to compress bunches. With respect to Fig. 1 achieving compression requires the cavity accelerating voltage to be falling as the bunch arrives hence the mode's phasor would be in the fourth quadrant.

In the absence of an LLRF system, or when an unwanted mode is damped and provided that bunches arrive continuously without gaps then a steady state voltage will become established for the unwanted mode. In this situation the phase advance and voltage damping between bunches is perfectly reset by the arrival of the next bunch. This pseudo steady state is synchronized to the bunch arrival times and not the mode frequency. This must be the case as the only drive frequency for the mode in the absence of a LLRF system is at the bunch frequency. The steady state mode vector prior to the arrival of a bunch and in the absence of RF control is derived in the next paragraph.

In the absence of beam loading the voltage $V(t)$ in a cavity evolves according to

$$\frac{d^2V}{dt^2} + \frac{\omega_c}{Q_L}\frac{dV}{dt} + \omega_c^2 V = 0 \tag{5}$$

where ω_c is the instantaneous cavity frequency and Q_L is the loaded Q factor. Letting the time between bunches be Δt_b then the change in cavity voltage between bunches is determined as $V \to Ve^z$, where

$$z = -\left[1 + j\sqrt{4Q_L^2 - 1}\,\right]\frac{\omega_c \Delta t_b}{2Q_L}. \tag{6}$$

Expressing the cavity voltage increment from a bunch determined from Eqs. (2) and (3) simply as δV then the condition for steady state is that $V(t) = V(t + \Delta t_b) = [V(t) + \delta V]e^z$. Solving $V = (V + \delta V)e^z$ gives

$$V = \frac{\delta V}{e^{-z} - 1}. \tag{7}$$

In Eq. (7) as before and without loss of generality the absolute phase of the kick can be chosen as zero so the phase of the cavity is determined by the term that multiplies δV. Defining

$$b = \frac{\omega_c \Delta t_b}{2Q_L} \tag{8}$$

and

$$\theta = \omega_c \Delta t_b \sqrt{1 - \frac{1}{4Q_L^2}} \tag{9}$$

then the phase of the cavity field at the instant before the bunch arrives is given by

$$\phi_V = -\tan^{-1}\left(\frac{\sin\theta}{\cos\theta - e^b}\right). \tag{10}$$

The magnitude at the same instant is determined as

$$|V| = \frac{|\delta V| e^{\frac{b}{2}}}{\sqrt{2(\cosh b - \cos\theta)}}. \tag{11}$$

Note that the steady state voltage does not depend on the starting voltage $V(0)$ or the relative phase of the first bunch. Fig. 2 plots the factor multiplying of δV in Eq. (11).

It is known [11] that the voltage in a mode only becomes large when the mode frequency is an integer multiple of the bunch frequency. For Fig. 2 these peaks are shown at 8, 9 and 10 times the higher bunch frequency of 40.08 MHz. For the compact 4 rod cavity design [6] the LOM has been positioned at 374 MHz but can be altered during design by a few MHz without affecting the performance of the operating mode. Fig. 2 shows that with a bunch frequency of 40.08 MHz then strong damping for the mode is unnecessary provided there is no chance of it shifting by 14 MHz to get to 360 MHz. For a bunch frequency of 20.04 MHz there are double the number of resonances with one occurring at 380 MHz. The requirement now becomes that the mode must not shift by 6 MHz. For a typical superconducting cavity such a large shift is impossible without a significant deformation of the cavity requiring a very large force. The cavity is designed to be sufficiently stiff for deformation from Lorentz force detuning to be less than 1 MHz. From Eq. (6) detuning due to loading is given as $f_0(1 - \sqrt{1 - 1/4Q_L^2})$ and for $Q_L \sim 100$ this gives a tiny shift of just 5 kHz. One remaining concern is detuning caused by mechanical deflection and deformation of the couplers and this requires further study.

For the LHC crab cavity, the voltage in the unwanted acceleration mode voltage will need to be kept very small at all times to meet stringent limits on the longitudinal impedance of 0.2 MΩ per cavity [12]. Typically this would be guaranteed by having a coupler that

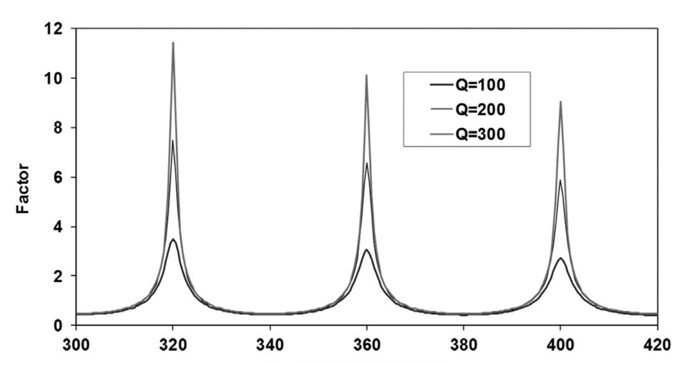

Fig. 2. Cavity voltage magnitude just before the bunch arrives as function of mode frequency.

strongly couples to the unwanted mode thereby extracting any power that the mode takes from the beam. Strong damping is only needed for mode frequencies close to a multiple of the bunch frequency. For most of the HOMs and potentially the LOM (lower order mode) there is an engineering uncertainty in the thermal contraction process and the tuning process with respect to frequency shifts. It is therefore necessary for all modes, unpredictable in this way (and which cannot be independently tuned), to be sufficiently damped. This means that for the LOM one needs testing and modeling to understand how its frequency might shift after manufacture during processing, cooling and then tuning of the operating mode.

With respect to establishing a controller to reduce or eliminate kicks from the accelerating mode it is useful to think about evolution of the cavity phasor as has been illustrated in Fig. 1. The phase reference is best referred to bunch arrival in which case $\alpha = 0$ in Eqs. (2) and (3) setting the voltage increment along the real axis. Eqs. (10) and (11) now give the cavity phasor the instant before the kick.

If the mode is resonant with bunch frequency then the starting phasor is on the positive real axis. For frequencies which are off resonance and for high loaded Q factors, the in-phase voltage before the bunch arrives tends to $-\delta V/2$ and goes to $+\delta V/2$ as the bunch passes through the cavity while the quadrature voltage can become significant when the bunches are not in anti-phase.

The steady state condition of Eqs. (10) and (11) becomes upset whenever there are missing bunches in the bunch train. The LHC has a lot of missing bunches, there are small gaps of 8 missing bunches associated with filling the SPS from the PS. There are larger gaps of either 38 or 39 bunches associated with filling the LHC from the SPS. Finally there is a very large gap of 102 bunches which is required for dumping the LHC beam.

Ordinarily after a gap, bunches get kicks that are substantially different from the kicks they would receive at steady state. Fig. 3 shows successive voltage kicks for bunches arriving 24.95 ns apart. A bunch train finishes at 28 μs, this is followed by a gap of 38 bunches (~ 1 μs), then a train of 72 bunches, then a gap of 8 bunches (~ 200 ns) then a new train.

These results are from a simulation that integrates the envelope equations[1] but incorporates all the details of the LHC bunch structure. In this case the LOM frequency was 374.08 MHz, its R/Q was 124.4 Ω, its external Q factor was 100 and the bunch charge was 32 nC. The intrinsic Q factor, Q_0 for superconducting cavities is invariably well over 10^6 hence the loaded Q factor can be regarded as being the same as the external Q factor throughout this paper. In Fig. 3 the first 5 voltage kicks after the long gap are −2539 V, −463 V, 717 V, −1315 V and −458 V; the settling value is −451.4 V.

Beam power extracted by the crab cavities has to be added again by the acceleration cavities. As 12 crab cavities might be needed on each beam then the acceleration cavities must replace about 450 V of lost voltage per bunch due to the LOM. For an LHC current of 1A this amounts to 450 W. Clearly the mode couplers on each of the crab cavities in this case need to extract this amount of power.

Fig. 4 shows simulated results for voltage in the cavity's unwanted acceleration mode corresponding to a train of 72 bunches after a gap of 38 bunches and followed by a gap of 8 bunches. When the mode has no initial voltage then a bunch charge of 32 nC then will excite an initial voltage of 4678 V as would be expected from knowledge of the R/Q, the bunch charge and the mode frequency. The fine structure in Fig. 4 is the exponential decay of the field after each bunch.

A problem with having differing kicks for different bunches is that where the main RF system is unable to respond sufficiently quickly to individual bunches then displaced bunches will not be at the optimum phase for acceleration and consequentially will have increased losses. Initially the losses will be from the leading bunches. Once charge is lost

[1] See appendix.

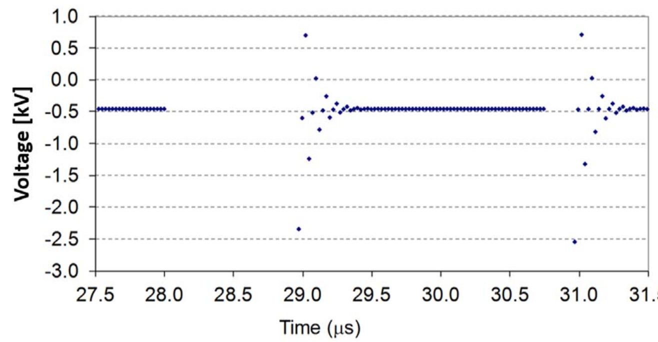

Fig. 3. Voltage kicks to successive bunches $Q_e = 100$ with no active control.

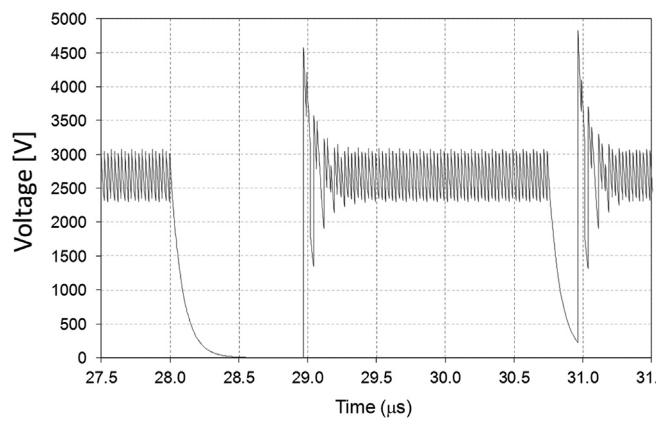

Fig. 4. Cavity mode voltage with no active control, $Q_e = 100$.

from the leading bunches the effective gap become larger and uneven kicks are then applied to bunches coming later in the train.

In Section 3 two opportunities offered by active damping are considered. Firstly, to control the amplitude and phase of the unwanted acceleration mode so it is at the steady state point whenever a bunch arrives thereby compensating for gaps in the bunch train. Good compensation can be achieved even with very small amounts of power. Secondly, to use RF power to move the in phase voltage back to $-\delta V/2$ whilst maintaining the quadrature voltage at the steady state point. This strategy ensures every bunch gets zero net kick. The amount of power required depends on how far the steady state point is from $-\delta V/2$.

3. Control strategy

An idealized LLRF system that might be used for active damping of an unwanted mode is shown in Fig. 5.

The RF system needed to drive the mode needs to operate close to the mode's natural frequency so as to minimize power usage. Overall excitation of an unwanted mode is always at a frequency close to a harmonic of the bunch repetition frequency. This is composed of a driven oscillation near to the unwanted mode frequency plus potentially large phase jumps caused by bunches that moves the mode phase advance close to a multiple of 2π with respect to the bunch frequency. Active damping can be applied for any frequency of the unwanted mode with a conventional LLRF system. When the mode frequency differs from the bunch excitation frequency and the RF oscillator is centered on the mode frequency then active damping requires a new set point to be determined after each bunch has passed through the cavity. Effectively the LLRF system has to acknowledge that part of the phase advance per cycle is being provided by the beam. Stated another way, when a bunch passes though the cavity there is a jump in the phase. If the RF system driving the mode to a set voltage at the instant of each

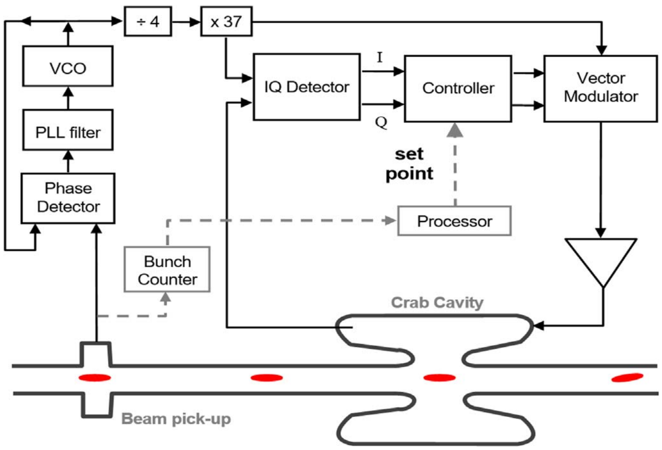

Fig. 5. LLRF system controlling a cavity mode.

bunch has provided the correct amplitude and phase then the error term that corrects the RF after the bunch needs to remain almost the same. The jumps in the set points are just following expected phase changes caused by bunches. The set point is an IQ vector. Each new set point is calculated by a simple vector addition after each bunch has passed through the cavity based on the best estimate for the mode phase. The nominal vector change for the set point is calculated from the bunch repetition frequency and the best estimate for the mode frequency. Because the mode is heavily damped control is relatively insensitive to errors in estimating the mode frequency.

The set point for the RF system is set after each bunch according to the algorithm

$$A_r(i) = |V|\cos(\phi_V + \theta_i)A_i(i) = |V|\sin(\phi_V + \theta_i) \tag{12}$$

where $A_r(i)$ and $A_r(i)$ are the in-phase and quadrature set point voltages for the mode with respect to the synthesized clock. $|V|$ and ϕ_V are the steady state amplitude and phase as defined previously in Eqs. (10) and (11) and θ_i is the expected RF phase for the next bunch.

For an arbitrary LOM frequency, there could potentially be an infinite number of set points, thus for clarity the simulations presented here use a LOM frequency such that only 3 set points are required. This means that θ_i in Eq. (12) cycles through three values and the exact LOM frequency which provides 3 set points is 374.08 MHz. The RF oscillator does not need to be at the exact center frequency of the mode as the amplifier has a bandwidth and its precise frequency is determined by the controller correcting the phase, i.e. the vector modulator can add or subtract a frequency from the oscillator. The RF oscillator frequency for the unwanted mode would typically be generated from the bunch repetition frequency using an integer divide PLL. For the example frequency of 374.04 MHz, synthesis is by dividing the bunch repetition frequency of 40.08 MHz by 3 and then multiplying by 28. Locking the drive frequency to a rational multiple of the bunch frequency forces the phasing between bunches and the LOM to maintain a predictable advance. In this case the LOM does nine and a third cycles between bunches hence the set points cycle after three bunches or 28 LOM cycles. The bunch phase is predictable from the main timing system and hence a dedicated beam pick up shown in the left hand side of Fig. 5 is unlikely to be needed; although it may be useful as a reference. The phase and amplitude of the unwanted LOM in the crab cavity is irrelevant except at the instant that bunches pass through the cavity. Here the amplitude and phase of the cavity would be measured with respect to the steady synthesized clock at 374.08 MHz.

Each new set point is chosen so that when the next bunch arrives in the cavity it either.

(a) has the steady state amplitude and phase

or

(b) has an amplitude and phase that gives zero bunch kick.

For a continuous train of bunches the set point moves by an amount almost equal to the amount that each bunch shifts the amplitude and phase of the mode. This means that for case (a) above the LLRF system does not need to deliver power unless there is a drift in the mode's natural frequency and for case (b) only a small amount of power is delivered. For a continuous bunch train the set point cycles increments by the same vector for each bunch, however when there is a gap in the bunch train the next set point depends on the number of missing bunches. The nature of the controller shown in Fig. 5 must be chosen with respect to the timescale over which corrections must be made. If corrections are to be made on every bunch then the correction must be made in 25 ns. If the correction is to be made during the short gap of 8 bunches there is a period of 200 ns in which to make the correction. For an accelerator environment making feedback corrections for individual bunches in 25 ns is probably impossible. Analog corrections within 200 ns are possible but digital control on this timescale is challenging. For the damping of the unwanted acceleration mode, most of the control action would be driven as feed forward corrections by manipulation of the set point vector additions. During an 8 bunch gap the controller needs to rotate the cavity phasor to a point near to where it should have been had the bunches not been missing. If the new set point is written to an analog controller as the last bunch enters, then given that the rotation is less than π a bandwidth of a few MHz is sufficient for the new set point to be achieved on the correct timescale. When set points are chosen optimally then feedback corrections become minimized. At the LHC the charge of every bunch would be known, its time of arrival in the cavity can be accurately predicted and hence its action on a low frequency accelerating mode can also be accurately predicted. In order to make a correction therefore the control system must send a predetermined amount of charge into the cavity at the correct phase over a number of RF cycles to achieve each new set point. Variations in bunch charge and detuning of the mode would require an element of feedback.

Optimal algorithms for the feed forward controller and a methodology have yet to be developed. One simple method to determine the feed forward power is to use the results of a simulation employing a high gain proportional controller with no delays in its action. The power that it predicts would then be the power that is used in the real controller. Of course one still needs accurate synchronization for the application of this power. As the unwanted acceleration mode is certain to have a very low external Q factor then feedback to compensate for frequency drift of the mode is not critical in the way that it would be for the operating mode in a typical accelerating cavity. The analysis in the following section uses a high gain proportional controller (no integral term) with minimal delay. When the drive power that this controller predicts is regarded as the input to the real cavity then the mode amplitude, the mode phase and bunch kicks would be nominally the same as the predictions. The feed forward term coming from the simulation is based on expected bunch charge and mode center frequency. As some variation is expected, the feed forward contribution might be supplemented with a feedback term based on errors for the previous bunch train. The feedback period might be the 80 buckets associated with the PS fill, the 270/271 buckets associated with the SPS fill or an entire LHC train.

For these simulations the new control set point is given to the controller on the time iteration after the bunch has passed through the cavity. (The software that has been developed has the option to consider any delay greater or equal to one time iteration.) The time iteration chosen for the simulations was the period of the unwanted mode.

For a real system the set point is compared to a measured value of the cavity voltage. The measurement system which can be regarded as part of the IQ detector shown in Fig. 5 will have a bandwidth. The software includes a measurement bandwidth but as code is being used to determine the feed forward term the bandwidth has been set very high.

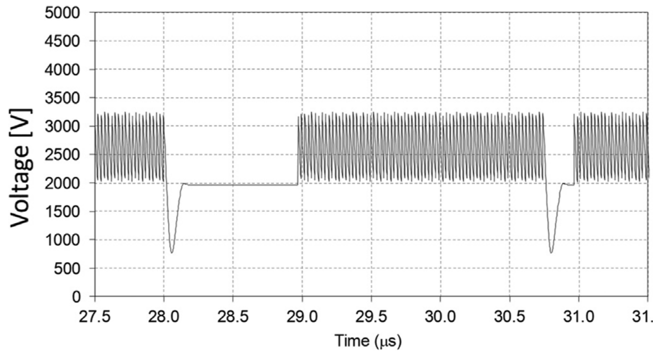

Fig. 6. Mode voltage using active control with gain=1500, Q_e=100, set point=no control steady state point.

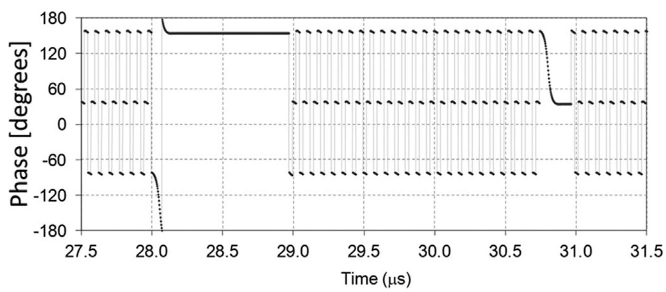

Fig. 7. Mode phase when bunch at cavity center using active control with gain=1500, Q_e=100, set point=no control steady state point.

4. Active damping performance

Figs. 6, 7, 8 and 9 plot computed mode voltage amplitude, phase and RF power and the voltage kick applied to the beam respectively for the proposed controller. The controller is fully feed forward, but the I and Q components of the drive are computed from a high gain proportional controller using cyclic set points to keep the amplitude and phase at the point to which they are naturally damped.

The slew rate of the amplifier is determined by the proportional gain and the amplifier bandwidth. The amplifier bandwidth was chosen as 50 MHz and the proportional gain taken sufficiently high for the new set point to be easily achieved within the short 8 bunch gap of 200 ns. Comparing Fig. 6 with Fig. 4 the voltage now starts in its steady state pattern for the bunch train. A voltage level is set during the gap of missing bunches to ensure that the cavity is at the correct amplitude and phase for the next bunch.

Fig. 7 shows the three phases associated with chosen frequency ratios. The phase is measured with respect to the master oscillator running at the center frequency of the LOM (phase with respect to bunch arrival times is of course tending to a constant value). In this particular case a phase of 155° is set during the long gap and a phase of 38.4° is set during the short gap in accordance with the expected time

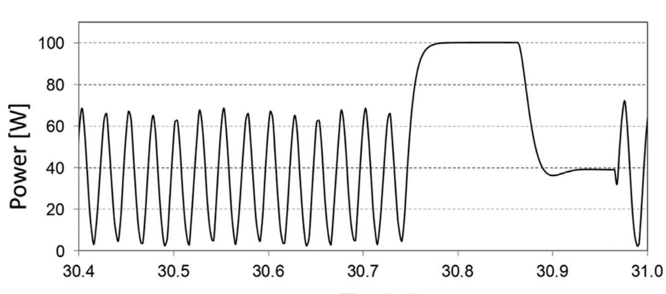

Fig. 8. RF power using active control with gain=1500, Q_e=100, set point=no control steady state point.

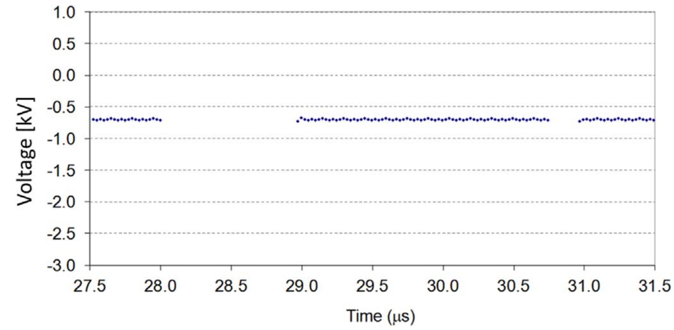

Fig. 9. Bunch kicks using active control with gain=1500, Q_e=100, set point=no control steady state point.

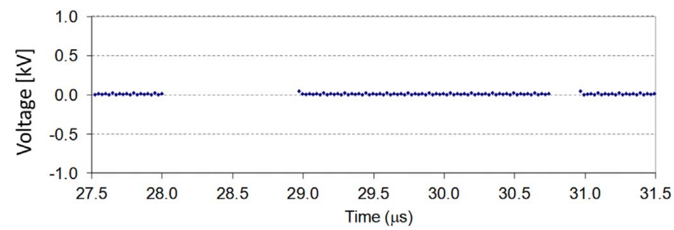

Fig. 10. Bunch kicks using active control with gain=1500, Q_e=100, set point chosen to give zero kick.

of arrival of the following bunch. For this simulation the maximum power was constrained to 100 W which is just below the peak demand from the controller during gaps.

Fig. 8 initially shows the required power towards the end of a train of 72 bunches. Close examination of the data indicates that bunches arrive as the power dips to zero. The last bunch in the train arrives at 30.74 μs. After 30.74 μs the figure shows the power used to reset and maintain a new amplitude and phase in anticipation of the next bunch during a short 8 bunch gap. The new level is achieved at 30.9 μs after which the power gets reduced to 40 W in order to maintain the set point. The figure shows the controller supplying power for each bunch when it should not be adding any (note that maintenance of the steady state point should not require power). A close comparison of Figs. 4 and 6 indicates that the set point is being over shot during the bunch train; even so almost exactly the same voltage is achieved in the mode for every bunch of the train.

Fig. 9 shows identical voltage kicks applied to successive bunches. The steady state voltage kicks are slightly higher than for the case with no active damping (Fig. 3) and this is because unnecessary power was supplied. The kicks can be reduced to zero by altering the set point voltage and allowing a higher power overhead to compensate the gaps. This case is shown in Fig. 10. Zero voltage kick was achieved with a set point voltage of 3400 V. In order to achieve the set point with the same gain as before the power limit for the amplifier was increased to 200 W and the amplifier bandwidth was increased from 50 MHz to 70 MHz.

Fig. 11 shows the power requirement for the voltage kicks associated with Fig. 10. The power requirement to achieve zero kicks is slightly higher than that shown in Fig. 8 where the intention had been to maintain the steady state point.

Control with minimal power during the bunch train can be obtained by reducing controller gain and amplifier bandwidth. Results when the gain is reduced by a factor of 5 and the amplifier bandwidth is reduced from 50 MHz to 15 MHz are shown in Figs. 12–15. Drive power is shown in Fig. 12, the first peak is at the start of a long gap of 38 missing bunches and the second peak is for a short gap of 8 missing bunches elsewhere during bunch trains the power is practically zero.

When the amplitude in Fig. 13 is compared with the amplitude in Fig. 6 it should be noted that Fig. 13 has its time axis expanded around

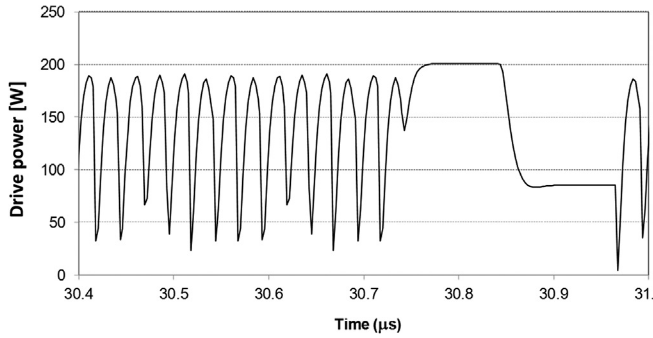

Fig. 11. Drive power required for zero kick.

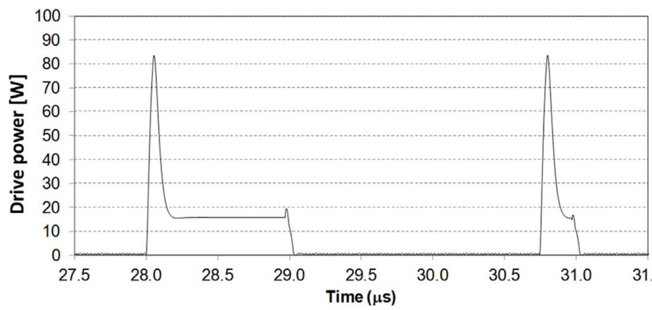

Fig. 12. RF power at gain=300, amplifier bandwidth=15 MHz, Q_e=100.

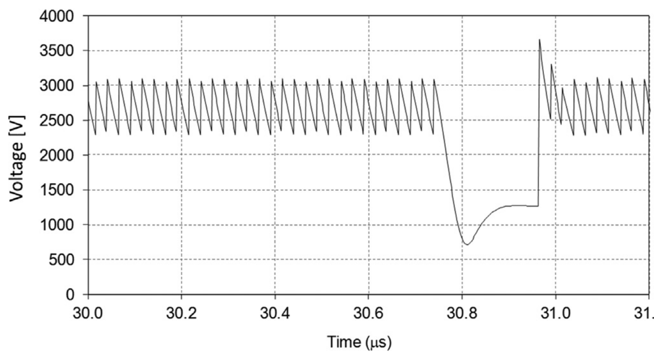

Fig. 13. Mode amplitude for gain=300, amplifier bandwidth=15 MHz, Q_e=100.

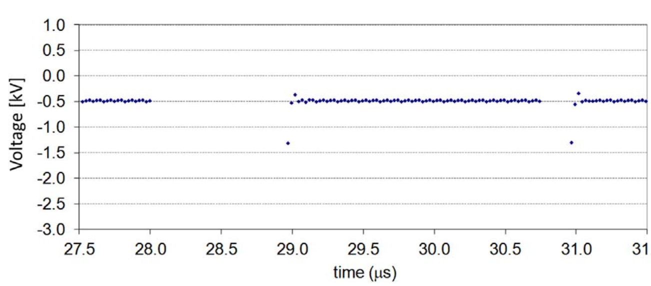

Fig. 14. Bunch kicks for gain=300, amplifier bandwidth=15 MHz, Q_e=100.

the short gap in the bunch train. During the bunch train Fig. 13 shows the variation in the mode voltage to be reduced with respect to Fig. 6, this is because no power is going into the mode. The variation in amplitude for Fig. 13 is now similar to the case without control shown in Fig. 4; except at the start of a train.

The resulting kicks shown in Fig. 14 are much reduced at the start of the train as compared with Fig. 3 but worse than in Fig. 10 where compensation was almost perfect. It is likely that an optimal control scheme can be found which only applies power to the first few bunches and achieves identical kicks for every bunch. The easiest way to

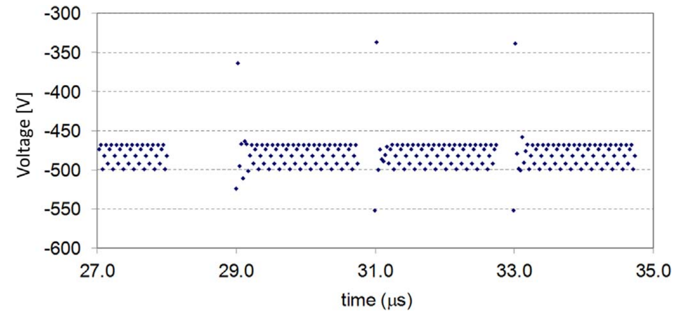

Fig. 15. Same as Fig. 14 but with expanded time axis to show the levels of voltage kick after transients have died away.

construct one is to reduce the gain during the bunch. It is of interest to show the kicks of Fig. 14 on an expanded scale (Fig. 15) which shows three distinct levels associated with the three phases.

Distinct levels arise whenever there are delays in the controller or averaging of measurements of the mode amplitude. Increasing the bandwidth for the measurements or increasing the integral term in the controller increases the splitting of these levels. As delays in the control system increase, the gain must be reduced to limit the splitting of these levels.

5. Active damping at resonance

When the acceleration mode is damped to a Q of 100 then the bandwidth of the mode is 3.7 MHz. During operation with a 25 ns bunch separation it is necessary that the mode never moves by 14–360.72 MHz. More critically during operation with a 50 ns bunch separation it is necessary that the mode never shifts by 6.68–380.76 MHz. It is desirable to reduce the damping of the acceleration mode by increasing the external Q factor from 100 to 300 or more to increase security against the mode ever being driven onto resonance. When the simulations of Section 4 are repeated for an external Q factor of 300 the RF power must be increased to about 300 W for a similar control performance. The average voltage in the cavity remains at 2.7 kV but with less variation. The set point can be fixed to give zero voltage kick.

If one now considers the worst case scenario with 25 ns bunch separation where the unwanted acceleration mode moves to 360.72 Hz it is shown later in this section that active damping can limit the cavity voltage and the voltage kicks to an acceptable level. For this case one no longer takes the set point voltage as the steady state voltage as determined by Eq. (11) as this is very high; instead a much smaller voltage is taken. For the simulation results presented in the following figures, Eq. (10) is used to provide the phase and the set point voltage is taken as 3140 V.

Figs. 16, 17 and 18 plot mode voltage amplitude, mode phase and RF power respectively, on resonance with active damping using the

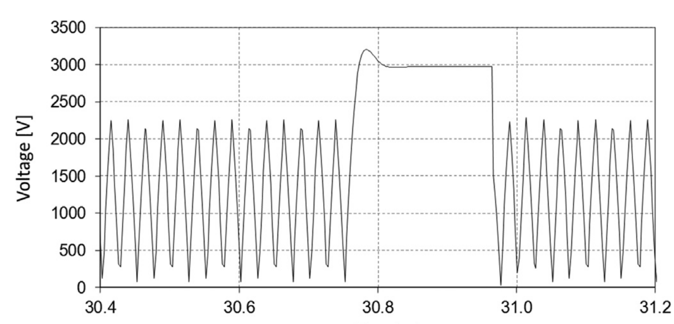

Fig. 16. Mode voltage on resonance for gain=1500, Q_e=300. note that data sampling is not able to show amplitude dips extending to zero on phase reversal.

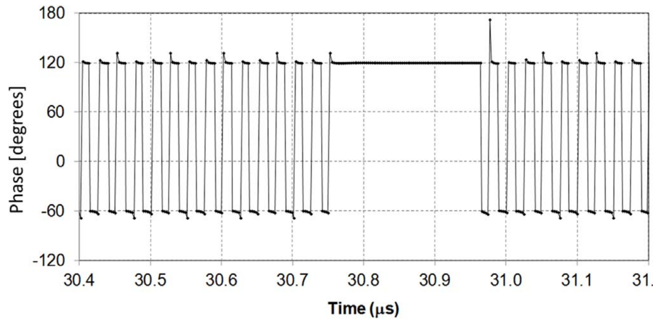

Fig. 17. Mode phase on resonance for gain=1500, Q_e=300.

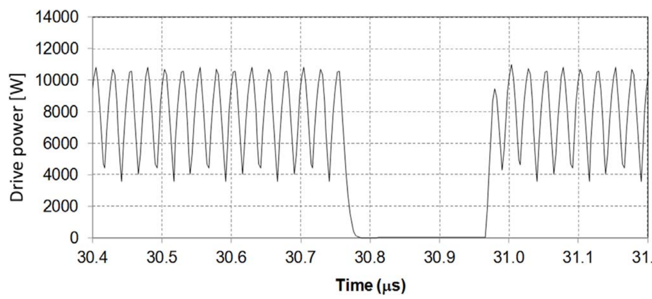

Fig. 18. RF power on resonance for gain=1500, Q_e=300.

same control parameters as used for the calculations presented in Figs. 6–10 of Section 4.

The power available from the amplifier was increased to 12.5 kW. In the absence of active control the mode voltage rises to 50 kV at the end of each bunch train and the peak power extracted from the beam is 30 kW. Other proposed crab cavity solutions for the LHC luminosity upgrade [8] (as opposed to the 4 rod cavity) would extract substantially higher power from the beam due to their higher monopole impedances. Figs. 16–19 show that with active control that the voltage flips from 2 kV with a phase of 120–2 kV with a phase of −60° when a bunch arrives (i.e. the voltage reverses). Fig. 16 shows amplitude, hence the flip at the voltage peak is not apparent. Power then drives the voltage back to its starting point and Fig. 17 shows a second phase reversal as the voltage passes through zero. Fig. 18 shows the power requirement for each bunch.

Fig. 19 shows that the worst voltage kick for the first bunch is only 700 V compared to 50 kV without compensation. Importantly only 11 kW peak power is required to achieve this control whereas 30 kW of peak power flows out of the coupler in the absence of active control.

With active control at resonance the waveform on the coupler is almost a standing wave hence power out almost equals power in. The 11 kW required for active control on resonance can be reduced to 4 kW for an external Q factor of 100 but needs to be increased to 35 kW for an external Q factor of 1000. Running at resonance is probably academic as one would expect to be able to tune the mode away from

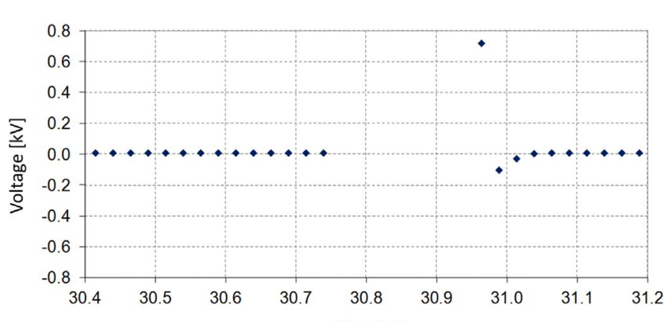

Fig. 19. Bunch kicks on resonance for gain=1500, Q_e=300.

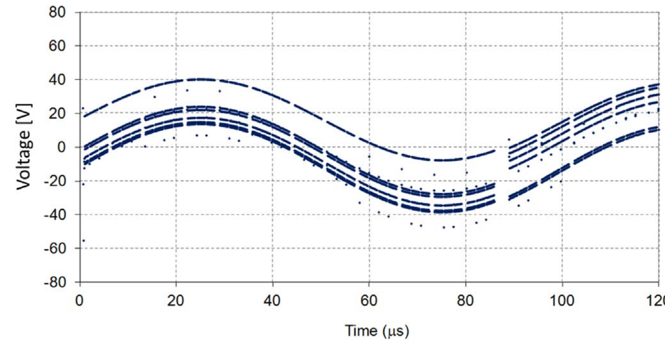

Fig. 20. Active control with microphonics.

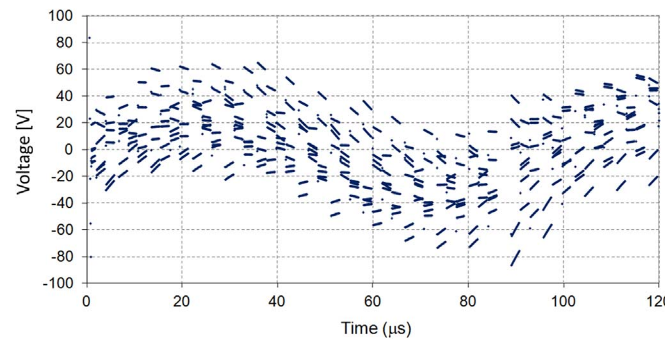

Fig. 21. Effect of introducing an integral term in the controller with respect to Fig. 20.

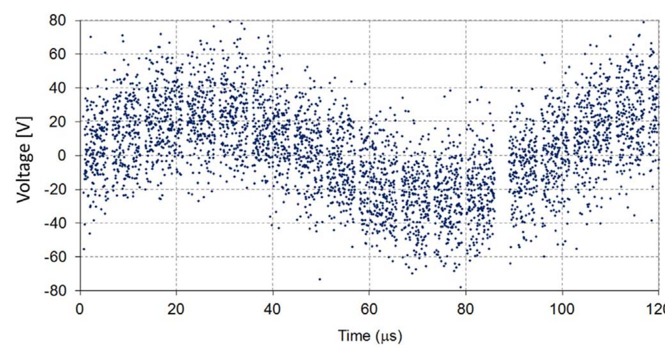

Fig. 22. Effect of introducing measurement errors with respect to Fig. 20.

resonance while warm during installation into the cryostat. This is not straightforward as sufficient testing on prototypes is required to understand frequency shifts of the LOM during cooling. It is important to realize that even at resonance the mode can be controlled with a modest amount of power for low external Q factors.

6. Mode detuning and measurement errors

An issue for superconducting cavities is control of phase and amplitude in the presence of microphonic detuning. The phase shift from detuning increases with loaded Q factor (Eq. (6)) hence when the loaded Q is low as would be the case here, then huge frequency shifts are needed before the effect upsets the control system. Fig. 20 shows kicks as a function of time when detuning with an amplitude of 200 kHz is introduced as a 10 kHz sinusoid. This amount of detuning would require a deflection of 0.1 mm to be applied to the cavity in its most sensitive dimension. Note that the time scale plotted is much longer than the periods used in previous figures hence many trains of 72 bunches are displayed.

The voltage axis scale is greatly expanded so that the splitting of the steady state previously observed in Fig. 15 can be seen. Detuning at the

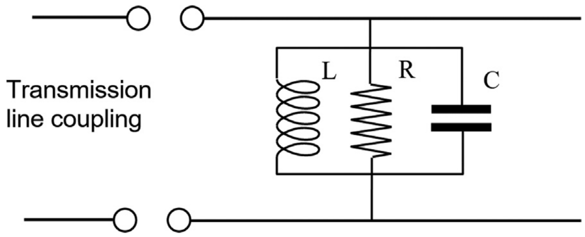

Fig. 23. Equivalent circuit of an RF cavity.

level of 200 kHz only perturbs the voltage kicks by ±40 V. In conventional LLRF control systems an integral term is introduced to eliminate tuning offsets. In this situation where the mode frequency is not an integer multiple of the bunch frequency an integral term gives no benefit to the controller. Fig. 21 shows the effect of introducing a moderate integral term into the controller; resulting in a randomization of the net kick to each bunch. Large integral terms always result in larger voltage kicks to bunches at the start of a train.

A key question for setting up the control system is the accuracy of measurement of amplitude and phase required for the unwanted LOM. Fig. 22 repeats the simulation of Fig. 20 with random phase and amplitude errors on the mode measurements. Specifically the phase error is taken as ±5° and the amplitude error as ±5%. The figure shows that even with huge measurement errors the random kicks are very small compared to the situation without active damping.

It is apparent in this system that performance is insensitive to measurement errors at a level significantly higher than one would normally expect for an accelerator system.

For the pure feed forward algorithm measurements are not needed once the charge in every bunch is known and one has a clock that is synchronous with the bunches, this is unless the mode frequency has shifted by a sizable fraction of its bandwidth. If an element of feedback is to be included as security against large frequency shifts one might directly sample the voltage in the mode with 8 bit accuracy at several GHz.

7. Conclusions

This paper sets out a hitherto unexplored method using active damping to eliminate wakes from low order acceleration modes in dipole cavities; this could be for mode frequencies above or below the dipole operating mode. Control would need to be primarily by feed forward. A method for determining the feed forward drive power has been set out and performance with respect to minimizing momentum kicks has been determined. The simulations have encompassed the complex LHC bunch structure and detuning. The paper shows that only a few hundred Watts of power is sufficient to eliminate the wake when the unwanted mode is far from resonance. In the event of a catastrophe moving the mode onto resonance then 11 kW of power is required to eliminate the wake when the loaded Q factor is 300.

It should be noted that to damp multiple modes, a controller is required for each additional mode, but corrective power can be supplied by a single broadband amplifier.

This work has been supported by STFC ST/G008248/1 and the European Union 7th Framework Program, grant number 227579 EuCARD.

Appendix A. Simulation model

The frequency separation of the unwanted acceleration mode from the dipole operating mode allows it to be modeled as a single LCR oscillator as shown in Fig. 23 where the transmission line is the coupler used to damp the mode. At the terminal, the voltage in the transmission line of the coupler must equal the voltage in the lumped circuit. Along the entry transmission line (i.e. the power coupler) the voltage and current satisfies the wave equation.

The current on the transmission line is given as

$$I(z,\,t) = \frac{1}{Z_{wg}}[V_F e^{j(kz-\omega t)} - V_R e^{-j(kz+\omega t)}] \tag{13}$$

where $k = \omega\sqrt{L_{wg}C_{wg}}$, $Z_{wg} = \sqrt{\frac{L_{wg}}{C_{wg}}}$, C_{wg} is the capacitance per unit length, L_{wg} is the inductance per unit length, V_F and V_R are the amplitudes of the forward and reflected voltage waves.

Taking the terminal between the cavity and the waveguide at z=0 and the voltage in the cavity as V then

$$V = (V_F + V_R)e^{-j\omega t}. \tag{14}$$

The current in the transmission line equals the sum of the currents through the equivalent circuit components of each series resonator hence

$$\frac{1}{L_{wg}}\int V dt + C_{wg}\frac{dV}{dt} + \frac{V}{R} = \frac{V_F - V_R}{Z_{wg}}e^{-j\omega t}. \tag{15}$$

By substituting Eq. (14) into Eq. (15), one can eliminate the reflected voltage and obtain

$$\frac{1}{L_{wg}}\int V dt + C_{wg}\frac{dV}{dt} + \frac{V}{R} + \frac{V}{Z_{wg}} = \frac{2V_F}{Z_{wg}}e^{-j\omega t}. \tag{16}$$

This equation determines the modal voltages in the cavity as a function of the amplitude of the forward wave in the waveguide. Defining the natural frequency of the mode as $\omega_0 = 1/\sqrt{L_{wg}C_{wg}}$ then the definition of the intrinsic and external Q factors gives $Q_0 = \omega R C_{wg}$ and $Q_e = \omega Z_{wg} C_{wg}$ respectively hence

$$Z_{wg} = \left(\frac{R}{Q_0}\right)_C Q_e. \tag{17}$$

The suffix C is used to denote the circuit definition of R/Q. Defining the loaded Q factor using

$$\frac{1}{Q_L} = \frac{1}{Q_0} + \frac{1}{Q_e} \tag{18}$$

then differentiation of Eq. (16) with the given definitions give the driven cavity equation as

$$\frac{d^2V}{dt^2} + \frac{\omega}{Q_L}\frac{dV}{dt} + \omega_0^2 V = \frac{2\omega}{Q_e}\frac{d}{dt}\{V_F e^{-j\omega t}\}. \tag{19}$$

In this equation ω is the RF drive frequency and ω_0 is the angular frequency for the mode in a lossless cavity.

For resonant systems where Q factors are greater than about 30 one does not need to solve for the voltages at any instant, it is sufficient to solve for the amplitude and phase. More conveniently than solving for amplitude and phase one solves for in phase and quadrature components of the voltage. Here the in phase part is denoted with the suffix r and the quadrature path with the suffix i. The in phase and quadrature voltages A_r and A_i can be defined with respect to the RF master oscillator frequency ω as

$$V(t) = (A_r(t) + jA_i(t))e^{-j\omega t}. \tag{20}$$

After making approximations consistent with a slowly varying amplitude and phase, Eq. (19) can be replaced with the two first order differential equations as follows

$$\left[\left(\frac{2\omega}{\omega_0}\right)^2 + \left(\frac{1}{Q_L}\right)^2\right]\frac{1}{\omega_0}\dot{A}_r + \left(\frac{\omega^2}{\omega_0^2} + 1\right)\frac{1}{Q_L}A_r + \left[\left(\frac{1}{Q_L}\right)^2 - 2\left(\frac{\omega_0^2 - \omega^2}{\omega_0^2}\right)\right]\frac{\omega}{\omega_0}A_i = \frac{2}{Q_eQ_L}\left(\frac{1}{\omega_0}\dot{V}_{F,r} + \frac{\omega}{\omega_0}V_{F,i}\right) + \frac{4}{Q_e}\frac{\omega}{\omega_0}\left(\frac{1}{\omega_0}\dot{V}_{F,i} - \frac{\omega}{\omega_0}V_{F,r}\right) \tag{21}$$

$$\left[\left(\frac{2\omega}{\omega_0}\right)^2 + \left(\frac{1}{Q_L}\right)^2\right]\frac{1}{\omega_0}\dot{A}_i + \left(\frac{\omega^2}{\omega_0^2} + 1\right)\frac{1}{Q_L}A_i - \left[\left(\frac{1}{Q_L}\right)^2 - 2\left(\frac{\omega_0^2 - \omega^2}{\omega_0^2}\right)\right]\frac{\omega}{\omega_0}A_r = \frac{2}{Q_eQ_L}\left(\frac{1}{\omega_0}\dot{V}_{F,i} - \frac{\omega}{\omega_0}V_{F,r}\right) + \frac{4}{Q_e}\frac{\omega}{\omega_0}\left(\frac{1}{\omega_0}\dot{V}_{F,r} + \frac{\omega}{\omega_0}V_{F,i}\right). \tag{22}$$

The difference between solving Eq. (19) and the envelope equations (Eqs. (21) and (22)) is that one no longer needs multiple time steps per RF cycle.

Beam loading is incorporated by allowing the phase and amplitude of the cavity excitation to jump in proportion to the image charge deposited in the cavity after the passage of the bunch see Eqs. (2) and (3) in the main text.

A digital LLRF system typically measures in phase and quadrature components of the cavity fields and controls each component to a set point by varying the in phase and quadrature components of the RF input. Importantly the system is described by two first order differential equations rather than one second order differential system. The optimum controller for a first order system with random disturbances is a Proportional Integral (PI) controller. The code used here has a PI controller option but the integral term is not used for the reasons given in the main text. When disturbances are well understood better controllers can be devised.

For any cavity mode an issue with the control is whether one can determine its amplitude and phase. If one can and with reference to the envelope equations one determines the drive for a PI controller as

$$V_{F,r}(t + t_{delay}) = c_p(A_{r,sp} - A_r) + c_i\left(\frac{\omega}{2\pi}\right)\int_{-\infty}^{t}(A_{r,sp} - A_r)dt V_{F,i}(t + t_{delay}) = c_p(A_{i,sp} - A_i) + c_i\left(\frac{\omega}{2\pi}\right)\int_{-\infty}^{t}(A_{i,sp} - A_i)dt \tag{23}$$

where t_{delay} is the time it takes to measure the error and adjust the amplifier output, $A_{r,sp}$ and $A_{i,sp}$ are the in phase and quadrature voltage set points and c_p and c_i are the gain coefficients for the proportional and integral controllers respectively.

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